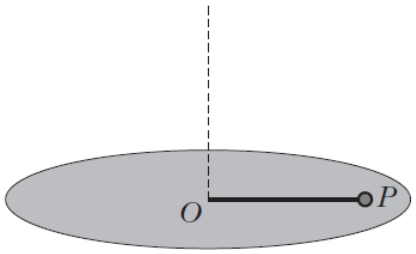
Hooke's Law & Horizontal Circles

2013

A6. A rough disc rotates in a horizontal plane with a constant angular velocity ω about a fixed vertical axis through the centre O. A particle of mass m kilograms lies at a point P on the disc and is attached to the axis by a light elastic string OP of natural length a metres and modulus of elasticity $2mg$.



The particle is at a distance of $\frac{5a}{4}$ from the axis and the coefficient of friction between P and the disc is $\frac{3}{20}$.

Find the range of values for ω such that the particle remains stationary on the disc. **5**

2006

A6. A conical pendulum consists of a bobbin of mass m kilograms attached to one end, B, of a light elastic string AB of natural length l metres and modulus of elasticity $8mg$ newtons. The other end, A, of the string is held fixed. The bobbin moves in a horizontal circle with centre vertically below A, such that the angle between the string AB and the vertical is $45°$.

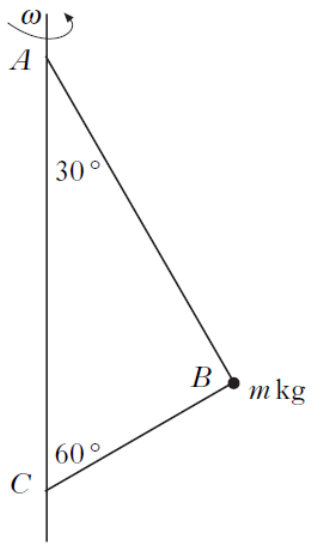
(*a*) Determine, in terms of l, the extension of the string beyond its natural length. **3**

(*b*) Show that the angular speed, ω radians per second, of the bobbin is given by

$$\omega^2 = \frac{8g}{(1+4\sqrt{2})l}.$$ **3**

2004

C8. A bead of mass m kilograms is attached to a vertical rotating column by two strings, as shown below. String AB is elastic, with natural length L metres and modulus of elasticity $2mg$ newtons. The string is attached to the column at A and to the bead at B. String BC is inextensible and has length L metres. The vertical column is rotating at ω rad s^{-1}, such that the strings AB and BC are taut and remain in a vertical plane. Angles ACB and BAC are $60°$ and $30°$ respectively.



(*a*) Show that the tension in the string AB is $2(\sqrt{3}-1)\,mg$ newtons. **4**

(*b*) Find, in terms of m and g, an expression for the tension in the string BC. **3**

(*c*) Given that $L = 1$, calculate ω. **4**